LIMITATIONS ON THE EQUIVALENT CARDIAC GENERATOR

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ABSTRACT This paper is a critical summary of the implications of potential theory in electrocardiography with particular attention to the "inverse problem." It is emphasized that potential measurements on the human torso serve to determine only the multipole coefficients of the heart sources and that no additional information is available. An interpretation of these coefficients with respect to the actual electrophysiological sources is considered further on the basis of distribution theory.

INTRODUCTION

Electrocardiography is concerned with a determination of the state of health of the heart from potential measurements made on the surface of the torso.[1] Research related to this problem can be considered as falling into one of three areas. These are summarized below.

The Direct Problem. Description of the electrical sources in the heart. This description may be in terms of the actual current generators in the myocardium, equivalent surface sources, or equivalent cardiac generators (Geselowitz, 1963) such as dipole, multiple dipole, (Gabor and Nelson, 1954) moving dipole, multipole, (Yeh et al., 1958; Geselowitz, 1960) and possibly others.

Determination of the potential field set up at the torso surface due to an elemental source (usually a current dipole) located at an arbitrary point within the torso (Green's function). Solutions of this aspect of the problem include those determined analytically for idealized geometries (Bayley and Berry, 1964) such as bounded cylindrical, spherical, ellipsoidal, etc. conducting bodies. In addition, there are available experimental solutions based on electrolytic models that take into account the torso shape and those that also include effects of inhomogeneities.

The Inverse Problem. This is the reverse of the direct problem and in-

[1] It is possible (though difficult) to measure cardiac potentials within the torso and also within the heart but such a procedure is obviously of a different character from conventional electrocardiography. Comments in this paper are restricted to electrocardiagram systems that acquire data from the body surface only.

volves the quantitative specification of the parameters of the electrical sources of the heart from potential measurements over the torso.

Diagnosis. The final step is the determination of the pathological state of the heart from a temporal description of the behavior of the electrical sources of the heart as found under *The Inverse Problem.*

THE DIRECT PROBLEM

The direct problem as outlined above has received considerable attention with the hope that such a study would lead to a better formulation of the inverse problem. Efforts have been directed to finding a suitable model for the effective sources of the heart and the surrounding conducting medium that is capable of an accurate simulation of the corresponding electrocardiogram.

The commonly used model is that of a single dipole fixed in space but variable as to orientation and magnitude and this constitutes the basis of current vectorcardiographic practice. This model is too simple since it destroys all information concerning the spatial distribution of the elemental sources. That is, for the ideal orthogonal lead field, the contribution from say an x-directed dipole element is independent of its position within the heart.

More recently, investigators have proposed other models of greater complexity. These include multiple dipoles, (Gabor and Nelson, 1954; Fischmann and Barber, 1953; Bellman et al., 1964; also Selvester et al., 1965) a single moving dipole, and a summation of multipoles (Yeh et al., 1958; Geselowitz, 1960). Other systems include double layer surfaces (Frank, 1953) either of an equivalent kind, (perhaps fixed in position) or directly related to the actual membrane generators (Gelernter and Swihart, 1964; McFee and Johnston, 1954).

Of the previous suggestions, the multiple dipole model is very attractive. In this case, a finite number of dipoles are chosen and arbitrarily located at significant sites throughout the heart. It is presumed that each dipole represents the average net electrical activity in its vicinity. If the correct values can be assigned these dipoles in the inverse problem, then information concerning discrete areas of the heart would become available, based on data obtained at the body surface. Such an accomplishment would surely be of great assistance in diagnosis of heart disease.

The single moving dipole model is somewhat less dramatic. However, here too, the position of the dipole is suggestive of the location of major electrical activity. This kind of information, as before, suggests improved diagnostic capability.

Of course, the richest description of the sources would be in terms of a detailed spatial and temporal account of the generator elements. Such a picture would not only carry information su h as reported by Scher (1956) on the pathways of activation but would also include the quantitative details of the vector source quantities within the active region. The genesis of the electrocardiogram, given such a source description, has been considered by Gelernter and Swihart (1964).

In formulating an electrical model of the heart, it is important to consider whether its parameters are uniquely specified by the surface potentials. If this is not the case then such a model cannot be used in a practical ECG system since all data are assumed to be restricted to that obtained from the body surface. A consideration of the direct problem is useful in *electrocardiography*[2] only when the assumed equivalent source is theoretically capable of quantitative specification in the inverse problem. Thus an additional condition must be included under *The Direct Problem* namely that only those electrical models of the heart which can be realized unambiguously from surface data are acceptable. We will show in the next section that because of this requirement the choice of an equivalent cardiac source is limited to the multipole representation.

THE INVERSE PROBLEM

Consider the true electrical sources of the heart at some instant of time. These sources can be described by the source function ω where

$$\nabla \cdot \mathbf{J} = \omega \tag{1}$$

and where $\mathbf{J}$ is the current density. The sources are depicted as distributed through a finite "heart region" in Fig. 1. The extent of the region is such that it can be contained, we assume, within some spherical surface of radius R_0. The conducting region in Fig. 1 is considered, for the present, as extending to infinity.

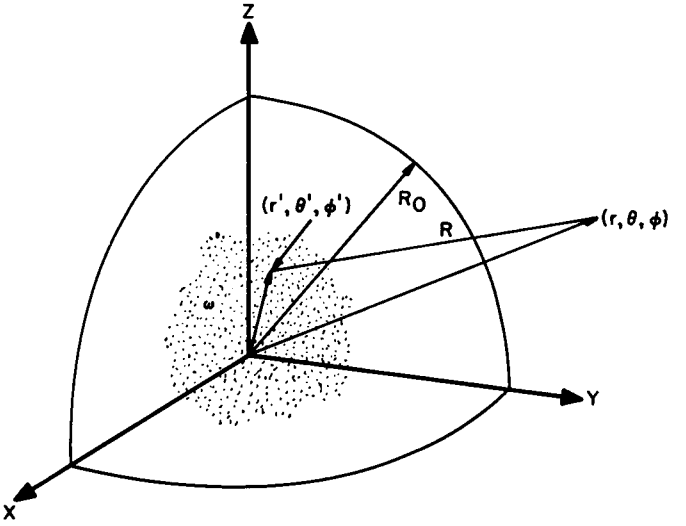

FIGURE 1 ECG system geometry.

[2] A consideration of the direct problem might be of interest as part of a study of the electrophysiology of the heart which is quite apart from the problem under discussion here.

Using Ohm's law, we have

$$\mathbf{J} = -\sigma \nabla \Phi \tag{2}$$

where σ is the conductivity and Φ the electric potential. Combining equations (2) and (1) yields the familiar Poisson's equation for Φ namely

$$\nabla^2 \Phi = -\omega/\sigma \tag{3}$$

The potential in the region external to the spherical surface of radius R_0 can be expressed as follows (Morse and Feshbach, 1953)

$$\Phi = \sum_{n=1}^{\infty} \sum_{m=0}^{n} [A_{nm} Y_{mn}^{e}(\theta, \phi) + B_{nm} Y_{mn}^{0}(\theta, \phi)] \frac{1}{r^{n+1}} \tag{4}$$

where

$$\begin{Bmatrix} A_{nm} \\ B_{nm} \end{Bmatrix} = \frac{\epsilon_m (n - m)!}{4\pi\sigma(n + m)!} \int_0^{2\pi} \begin{Bmatrix} \cos(m\phi') \\ \sin(m\phi') \end{Bmatrix} d\phi' \cdot \int_0^{\pi} P_n^m(\cos\theta') \sin\theta' \, d\theta' \int_0^{a} \omega(r', \theta', \phi') r'^{n+2} \, dr' \tag{5}$$

and

$$Y_{mn}^{e} = P_n^m(\cos\theta) \cos m\phi$$

$$Y_{mn}^{0} = P_n^m(\cos\theta) \sin m\phi$$

$$\epsilon_m = 1 \text{ for } m = 0, = 2 \quad \text{otherwise.}$$

Equation (4) is the multipole expansion for source. The order of the multipole is given by the value of n. Equation (5) shows how the multipole coefficients are related to the source. Since for bioelectric sources ω is made up of dipole elements one finds that each dipole component (A_{10}, A_{11}, B_{11}) is a simple algebraic sum of the corresponding component of all elemental dipole sources; i.e., the spatial distribution does not affect the resultant as noted earlier. The net dipole quantity corresponds precisely to the fixed position dipole of vectorcardiography.

The quadrupole term (as well as the higher terms) involve a spatial weighting of the source distribution. It is for this reason that these terms are likely to have considerable clinical significance. Since clinical vectorcardiographic practice utilizes the dipole term alone, an important task of current research should be the development of a suitable basis for the utilization of the higher terms in clinical practice if possible. One should note that except for the dipole, the multipole coefficients depend on the choice of origin. The effect of shifting the origin is to change the multipole coefficients in a way which depends only on the origin displacement; consequently there is fundamentally no modification of the information. However, it may be that a particular origin will result in a more rapid convergence and hence a simpler representation.

In the inverse problem if the potential is specified over some bounding surface, the

coefficients A_{nm} and B_{nm} may always be determined. In particular, if the potential Φ_S is measured over a spherical surface $r = R_0$ (but in an infinite medium) then the coefficients A_{nm} and B_{nm} are found by utilizing the orthogonality of the tesseral harmonics; one gets

$$\begin{Bmatrix} A_{nm} \\ B_{nm} \end{Bmatrix} = \frac{R_0^{n+1}}{2\pi} \left(\frac{2n+1}{\epsilon_m} \right) \cdot \frac{(n-m)!}{(n+m)!} \int_S \Phi_S P_n^m(\cos\theta) \begin{Bmatrix} \cos m\phi \\ \sin m\phi \end{Bmatrix} \sin\theta \, d\theta \, d\phi \qquad (6a)$$

With only a slight modification, a similar result is available for the finite spherical conducting region (Hlavin and Plonsey, 1963) namely

$$\begin{Bmatrix} A_{nm} \\ B_{nm} \end{Bmatrix} = \frac{R_0^{n+1}}{2\pi} \cdot \frac{n}{\epsilon_m} \frac{(n-m)!}{(n+m)!} \int_S \Phi_S P_n^m(\cos\theta) \begin{Bmatrix} \cos m\phi \\ \sin m\phi \end{Bmatrix} \sin\theta \, d\theta \, d\phi \qquad (6b)$$

For an arbitrary finite shape, the coefficients may be found by evaluating the following expression (Geselowitz, 1960)

$$\begin{Bmatrix} A_{nm} \\ B_{nm} \end{Bmatrix} = \frac{(n-m)!}{2\pi\epsilon_m(n+m)!} \int_S \Phi_S \nabla \left[r^n \begin{Bmatrix} \cos m\phi \\ \sin m\phi \end{Bmatrix} P_n^m(\cos\theta) \right] \cdot d\mathbf{S} \qquad (7)$$

where S is the arbitrary surface, and Φ_S is the measured surface potential.

Having found the multipole coefficients, we might now inquire as to whether this can be converted into information concerning the actual distribution of the source elements. It is known, however, that an infinite number of source distributions can have the same multipole expansion and could be implicated by measurements leading to a specific set of A_{nm} and B_{nm}. But in practice there is even a greater indeterminancy. This arises in the following way.

One notes from equation (4) that the higher the multipole order the larger the inverse power of r. This means that, in general, higher order multipoles contribute smaller amounts to the potential measurement. For a particular minimum detectable signal taking into account noise etc. then multipoles of order greater than, say, $n = N$ will be obscured.[3] This means that two sources with only a finite number of similar multipole terms may also appear equivalent.

The afore-mentioned value of N depends on the choice of origin, the particular source distribution, and the minimum distance to a surface point, as well as the minimum detectable signal. We designate by $N = N_c$ a conservative estimate in electrocardiography based on these factors, considering a choice of origin that is most likely (a priori) to assure convergence.[4] It should be noted that an equivalent

[3] ECG investigations based on factor analysis (Horan et al., 1964) suggest that at most eight significant parameters are involved in the human. This suggests that $N = 2$ (three dipole plus five quadrupole components) with present measurement techniques.

[4] Corresponding to a fixed source, a change in origin will, in general, change the number of significant multipole coefficients. The coefficients obtained for one origin can be converted to that of another by means of "shift" formula such as equation (17) in Geselowitz (1960). Thus two sets $\{A_{nm}, B_{nm}\}$ although different may be fully equivalent if they represent the same distribution considered with respect to different origins.

source must not only possess a multipole expansion whose terms A_{nm} and B_{nm} for $n \leq N_c$ equal the measured values, but must also satisfy the constraint that the net surface potential due to all higher moments be less than the rms noise. This imposes certain bounds on all A_{nm} and B_{nm} for $n > N_c$.

A multipole source consisting (only) of the measured values of A_{nm} and A_{nm} ($n \leq N_c$) will duplicate the entire surface potential to within detectable limits. Since the number of coefficients is M, where $M = \Sigma_{n=1}^{N_c} (2n + 1)$, only M surface potentials can be considered as independent. In a specific case it may be that $N < N_c$ since N_c is here defined as a conservative estimate on the entire class of ECG's.

A physical distribution designed to produce the same surface potential as is measured is often sought as an equivalent source. We see that such a distribution must itself possess precisely the multipole expansion as described above. As a nonrigorous way in insuring that moments beyond N_c be appropriately bounded we choose instead an $N = N_e \gg N_c$, where all coefficients A_{nm} and B_{nm} corresponding to $N_c < n \leq N_e$ are taken as zero. By means of this device we expect that any two different source distributions with identical multipole terms out to $n = N_e$ will produce similar surface potential fields. We are depending here on the usual circumstance where higher terms ($n > N_e$) produce negligible surface potential effects.

One class of physical sources of great interest is that of multiple dipoles with (perhaps) some spatial constraint as imposed by the choice of N_e. Let us assume that a fixed number, L, is distributed in some way in the heart. If the location is specified a priori then the orientation and magnitude of each dipole is at our disposal and can be adjusted to fit the measured surface data. But it *will* fit these data, within the limits of our ability to detect the signal, if the dipole distribution yields a multipole expansion that corresponds to the specified values to the N_e^{th} term. This can be accomplished by finding the multipole expansion of each dipole separately, superposing the corresponding contributions from each dipole to a specific multipole component and setting that sum equal to the required value (of that multipole component). We obtain in this way M equations[5] in $3L$ (each dipole is specified by its three rectangular components) unknowns. As long as $L \geq M/3$, a solution is available. And for $L > M/3$ the problem is overspecified and we can *arbitrarily fix* $(L - M/3)$ scalar dipole quantities.

Several consequences can now be considered. The first is that if we measure the potential field over the surface of a torso and conceive this field as due to an assembly of L dipoles, where $L > M/3$, then we can fit the measurements with a very wide variety of dipole distributions. The difficulty in seeking a multiple

[5] We assume that the spatial distribution of dipoles such that each multipole component is contributed to by at least one dipole element. Such would not be the case if, for example, the dipole elements were required to be on a straight line.

dipole representation, thus, is that it is not unambiguously specified. The wide variety of multiple dipole distributions that yield indistinguishable surface potential distributions, yet possess different physiological implicatons, underscores the incorrectness of seeking this type of interpretation. Fundamentally, the measurements have produced a set of multipole coefficients $\{A_{nm}, B_{nm}\}$, this represents the extent of the available information. Consequently all diagnostic values should be directly assignable without the need for further interpolations that cannot add information.

The consequence of a finite signal/noise ratio is that equivalent source distributions are determined by a finite (rather than an infinite) number of multipole terms (i.e. N_c or N_e). However, the redundancy in physical distributions would occur even if all orders of multipole coefficients could be specified (i.e. measured). Morse and Feshbach (1953), for example, illustrate this fact by finding a source distribution whose multipole coefficients all vanish. Another point of view of this question is developed in a different way in the next sections.

DISTRIBUTION FUNCTIONS

The previous discussion merely puts into a perspective appropriate to electrocardiography that surface potential measurements do not uniquely determine the volume source distribution. A graphic example is also to be found in Morse and Feshbach. We continue here to develop still another view, which is of interest since it defines what aspects of a source can be determined and what cannot.

We consider first an alternate expression for the potential distribution

$$\Phi(P) = \frac{1}{4\pi\sigma} \int_V \frac{\omega}{R}\, dV' \tag{8}$$

where the geometry of Fig. 1 is retained. Instead of expanding $1/R$ in tesseral harmonics, which leads to equations (4) and (5), one writes the Taylor series representation for R^{-1} about the origin $x' = y' = z' = 0$. This gives (Morse and Feshbach, 1953)

$$\Phi(P) = \sum_{n=0}^{\infty} \sum_{\ell=0}^{n} \sum_{k=0}^{n-\ell} \frac{(-1)^n}{\ell!\, k!\, (n-\ell-k)!} C_{nk\ell} \frac{\partial^n}{\partial x^\ell\, \partial y^k\, \partial z^{n-\ell-k}} \left(\frac{1}{r}\right) \tag{9}$$

$$C_{nk\ell} = \frac{1}{4\pi\sigma} \int_V \omega(x', y', z') x'^\ell y'^k z'^{(n-\ell-k)}\, dx'\, dy'\, dz' \tag{10}$$

The coefficients $C_{nk\ell}$ are recognized as the joint source distribution moment. We note that, as with equation (4), equation (9) is also an expansion in inverse powers of r. Thus it too is a multipole representation. However, in this case, the n^{th} multipole has $\frac{1}{2}(n+1)(n+2)$ components as contrasted to $(2n+1)$ for the tesseral harmonic form. Clearly, a redundancy of $[\frac{1}{2}(n+1)(n+2) - (2n+1)]$ exists; the

redundancy increases with increasing n, being equal to zero only for the dipole term.

If we define the characteristic function of the source distribution as

$$M(v_1, v_2, v_3) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \omega(x', y', z') \exp (jv_1x' + jv_2y' + jv_3z') \, dx' \, dy' \, dz' \quad (11)$$

then the joint moment $C_{nk\ell}$ can be found from

$$C_{nk\ell} = (-j)^n \frac{\partial^n M(v_1, v_2, v_3)}{\partial v_1^{\ell} \, \partial v_2^{k} \, \partial v_3^{(n-\ell-k)}} \Bigg|_{v_1=v_2=v_3=0} \quad (12)$$

Thus the joint moments serve to define the Taylor series expansion of the characteristic function about the origin ($v_1 = v_2 = v_3 = 0$). That is

$$M(v_1, v_2, v_3) = \sum_{n=0}^{\infty} \sum_{\ell=0}^{n} \sum_{k=0}^{n-\ell} \frac{(j)^n}{\ell! \; k! \; (n - \ell - k)!} C_{nk\ell} v_1^{\ell} v_2^{k} v_3^{(n-\ell-k)} \quad (13)$$

If we assume that equation (13) converges sufficiently rapidly then the source distribution may be found by taking an inverse transform of equation (11) namely

$$\omega(x', y', z') = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} M(v_1, v_2, v_3) \exp (-jv_1x' - jv_2y' - jv_3z') \, dv_1 \, dv_2 \, dv_3 \quad (14)$$

Since the source function, ω, is confined to a finite region it can be shown (Kendall, 1948) that equation (13) must converge. Consequently, the joint moments $C_{nk\ell}$ uniquely determine the source distribution.

The meaning of the above in electrocardiography is that the source distribution, or a reasonable approximation thereof, may be obtained if the coefficients $C_{nk\ell}$ can be found. Examination of equation (10) reveals how each such moment gives a different aspect of the source distribution and the above theory indicates that this can be adequate for a determination of ω. The difficulty, however, is that $C_{nk\ell}$ cannot be measured since a measurement of the n^{th} multipole will be capable only of specifying the $(2n + 1)$ non-redundant coefficients; the remaining $[\frac{1}{2}(n + 1)(n + 2) - (2n + 1)]$ terms are redundant. Consequently, measurements in electrocardiography are fundamentally limited to a determination of only certain weighted averages of the source distribution namely that given in equation (5). This is not a complete picture, one would prefer the moments of equation (11), but it is all that is available.

As a specific example, consider the quadrupole coefficients in equation (4) which are related to the sources by means of equation (5). A connection exists between this set and the corresponding $C_{2k\ell}$ quadrupole terms. We get

$$A_{20} = \frac{1}{2\pi\sigma} \int_0^{2\pi} \int_0^{\pi} \int_0^{a} \left[\frac{3 \cos^2 \theta - 1}{4}\right] \omega r^4 \sin \theta \, d\theta \, d\phi \, dr$$

$$= \frac{1}{4\pi\sigma} \int_V \frac{(2z^2 - x^2 - y^2)\omega \, dV}{2} = (2C_{200} - C_{202} - C_{220})/2$$

$$A_{21} = C_{201} \qquad B_{21} = C_{210}$$

$$A_{22} = \tfrac{1}{4}(C_{202} - C_{220}) \qquad B_{22} = \tfrac{1}{2}C_{211}$$

The above set of five equations involves the six desired (unknown) coefficients for the expansion (9). We note that certain of the C_{nkl} moments are actually realizable[6] from measurements of A_{nm} and B_{nm} but not enough equations are available to obtain the remainder.

One must reluctantly conclude, again, that the multipole expansion coefficients constitute all the information attainable in a surface potential measurement. While the n^{th} moment requires ½ $(n + 1)(n + 2)$ coefficients for complete specification and only $(2n + 1)$ are available, one nevertheless does obtain more and more aspects of the distribution by identifying as many of the A_{nm} and B_{nm} terms as possible. Presumably, improvements in diagnosis will follow from identification of additional multipole components since each is uniquely related to a particular distribution aspect. But a deterministic approach to ECG diagnosis in terms of actual distributions of electrical activity cannot be achieved.

Parenthetically, it might be noted that since all A_{nm} and B_{nm} can be obtained by surface measurements the correct potential field within the body (but outside a sphere that encloses all sources) can be obtained. This implies the possibility of synthesizing "phantom" point electrodes within the torso and exterior to a sphere enclosing the heart. Such an ability was demonstrated by McFee and Johnston, (1954) who give the details for the construction of such a lead field. From a practical standpoint only a finite number of multipole terms can be measured (or synthesized). Furthermore, the fundamental problem of lack of uniqueness is still present and the conclusion is again reached that only multipole coefficients of the source are attainable.

CONCLUSIONS

Current research in electrocardiography covers the three areas noted earlier. With respect to work concerned with the direct problem it does not seem likely that this will ever be useful in electrocardiography. Thus, while such knowledge of the effect of various conducting pathways in the activation process and the effect of inhomogeneities on the "field producing process" are of some interest in *electrophysiology,* the lack of uniqueness in the inverse problem severely limits the application of this information to *electrocardiography.* For clearly the goal of electrocardiography is in the search for abnormalities. The location and extent of inhomogeneities, the depolarization pathways, etc. are themselves the object of this investigation and knowledge of their *general* properties is unlikely to be of much help in the inverse problem. Indeed, the effect of inhomogeneities can (and to a certain extent, must) be formulated in terms of a more complicated effective source distribution in a homogeneous medium.

[6] If the details are considered, one discovers that in addition to C_{201}, C_{210}, C_{211}, the coefficient C_{311} can also be obtained (Plonsey, 1966).

To emphasize this latter part, we note that when the total region consists of an arbitrary ensemble of regions each of uniform, but different, conductivity then the medium may be considered as if it were homogeneous provided that induced sources are located at all interfaces. This approach is the one taken in Gelernter and Swihart (1964), where it is noted that if E_n is the applied field normal to the interface then the appropriate induced surface charge density ω_s is

$$\omega_s = \frac{2\epsilon E_n(\sigma_1 - \sigma_2)}{(\sigma_1 + \sigma_2)}$$

where σ_1, σ_2 are the respective conductivities of the adjoining regions. For the situation where σ is a continuous, rather than a discrete, function of position, the equivalent sources (Krohn, 1962) are

$$\omega = \nabla\sigma \cdot \nabla\Phi$$

The conclusion here is similar to that suggested by Geselowitz (1963) that the equivalent heart generator be defined with respect to a homogeneous medium.

With respect to the inverse problem, the need is to establish the highest multipole moment that can be detected. Here the limitations of signal/noise must be considered and ways of maximizing this ratio are important. For a given multipole component, an appropriate measurement scheme is then required. The qualities of reproduceability are most important and constitute the greatest challenge. This is because the higher multipole terms depend critically on the effective origin and this must be similarly located within the torso independently of the body shape.

Any emerging system must finally be tested in practice. This would require an organized evaluation with clinical data using conventional schemes or perhaps involving pattern recognition of an adaptive kind (Specht, 1964; Okajima et al., 1963).

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REFERENCES

BAYLEY, R. H., and BERRY, P. M., 1964, *IEEE Tr. Bio-Med. Electron,* **11,** 137.

BELLMAN, R., COLLIER, C., KAGIWADA, H., KALABA, R., and SELVESTER, R., 1964, *Commun. Assn. Computing Machinery,* **7,** 666.

BRODY, D. A., BRADSHAW, J. C., and EVANS, J. W., 1961, *IRE Inst. Radio Engrs., Trans. Bio-Med. Electron.,* **BME-8,** 139.

FISCHMANN, E. J. and BARBER, M. R., 1953, *Am. Heart J.,* **65,** 628.

FRANK, E., 1953, *Am. Heart J.,* **46,** 364.

GABOR, D., and NELSON, C. V., 1954, *J. Appl. Physics,* **25,** 413.

GELERNTER, H. L., and SWIHART, J. C., 1964, *Biophysic. J.*, **4**, 285.
GESELOWITZ, D. B., 1960, *Proc. IRE Inst. Radio Engrs.*, **48**, 75.
GESELOWITZ, D., 1963, *Biomedical Sciences Instrumentation,* New York, Plenum Press, **1**, 325.
HLAVIN, M., and PLONSEY, R., 1963, *IEEE Tr. Bio-Med. Electron,* **BME-10**, 98.
HORAN, L. G., FLOWERS, N. C., and BRODY, D. A., 1964, *Circulation Research,* **15**, 131.
KENDALL, M. G., 1948, Advanced Theory of Statistics, **1**, London, C. Griffin & Co.
KROHN, L. H., 1962, *Bull. Math. Biophysics,* **24**, 277.
MCFEE, R., and JOHNSTON, F. D., June 1954, *Circulation,* **9**, 868.
MORSE, P. M., and FESHBACH, H., 1963, Methods of Theoretical Physics, New York, McGraw-Hill, 1276.
OKAJIMA, M., STARK, L., WHIPPLE, G., and YASUI, S., July, 1963, *IEEE Tr. Bio-Med. Electron.*, **BME-10**, 106.
PLONSEY, R., 1966, *Bull. Math. Biophysics,* in press.
SELVESTER, R. H., COLLIER, C. R., and PEARSON, R. B., 1965, *Circulation,* **31**, 45.
SCHER, A. M., and YOUNG, A. C., 1956, *Circulation Research,* **4**, 461.
SPECHT, D. F., June, 1964, Systems Theory Laboratory, Stanford, California, Stanford Electronics Laboratory Technical Report No. 6763-1.
YEH, G. C., MARTINEK, J., and DE BEAUMONT, H., 1958, *Bull. Math. Biophysics,* **20**, 203.